Lamb Shift and Proton Radius Puzzle

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Tuesday Colloquium

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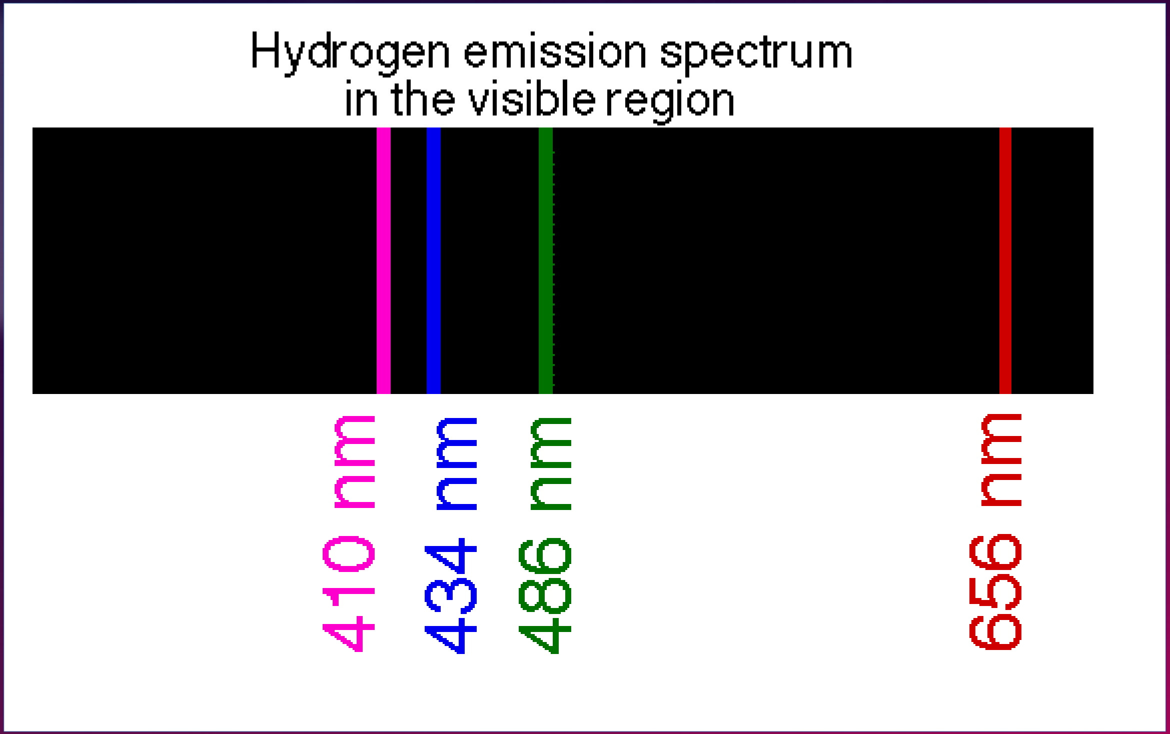
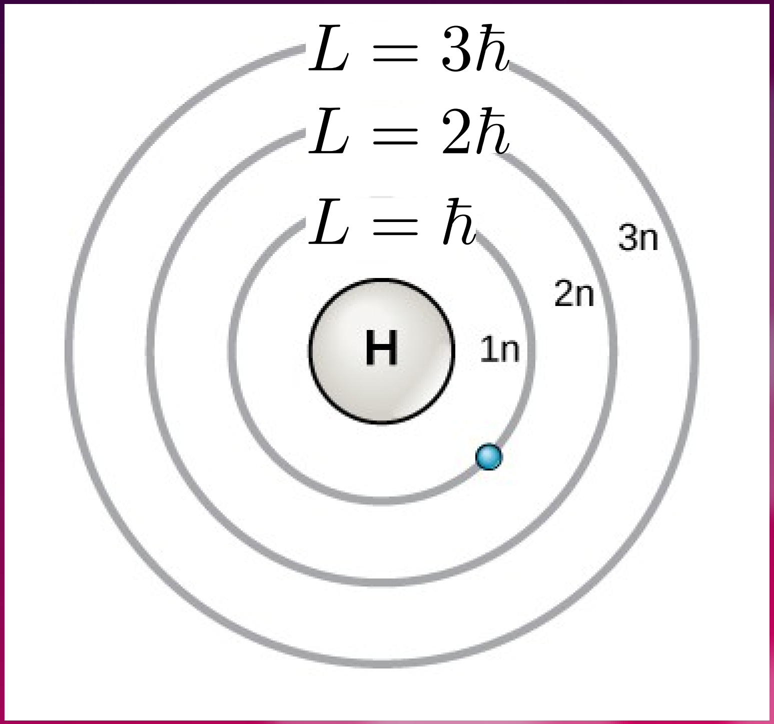
Hydrogen spectrum (1885)



Bohr (1913):



Quantum mechanical solution (1926)

$$\frac{-\hbar^2}{2m}\nabla^2\Psi(\vec{r}) + V(\vec{r})\Psi(\vec{r}) = E\Psi(\vec{r})$$

$$\text{Kinetic Energy} + \text{Potential Energy} = \text{Total Energy}$$

$$-\frac{\hbar^2}{\boxed{2m_r}}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]\Psi(r,\theta,\phi)$$
$$\boxed{-\frac{q^2}{4\pi\varepsilon_0 r}}\Psi(r,\theta,\phi) = E\Psi(r,\theta,\phi)$$

Reduced mass

- Make a 2-body problem into a 1-body one
- This will come back later in the talk

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

Therefore:

$$m_1 \mathbf{a}_1 = -m_2 \mathbf{a}_2 \quad \Rightarrow \quad \mathbf{a}_2 = -\frac{m_1}{m_2}\mathbf{a}_1$$

The relative acceleration $\mathbf{a}_{\text{rel}}$ between the two bodies is given by:

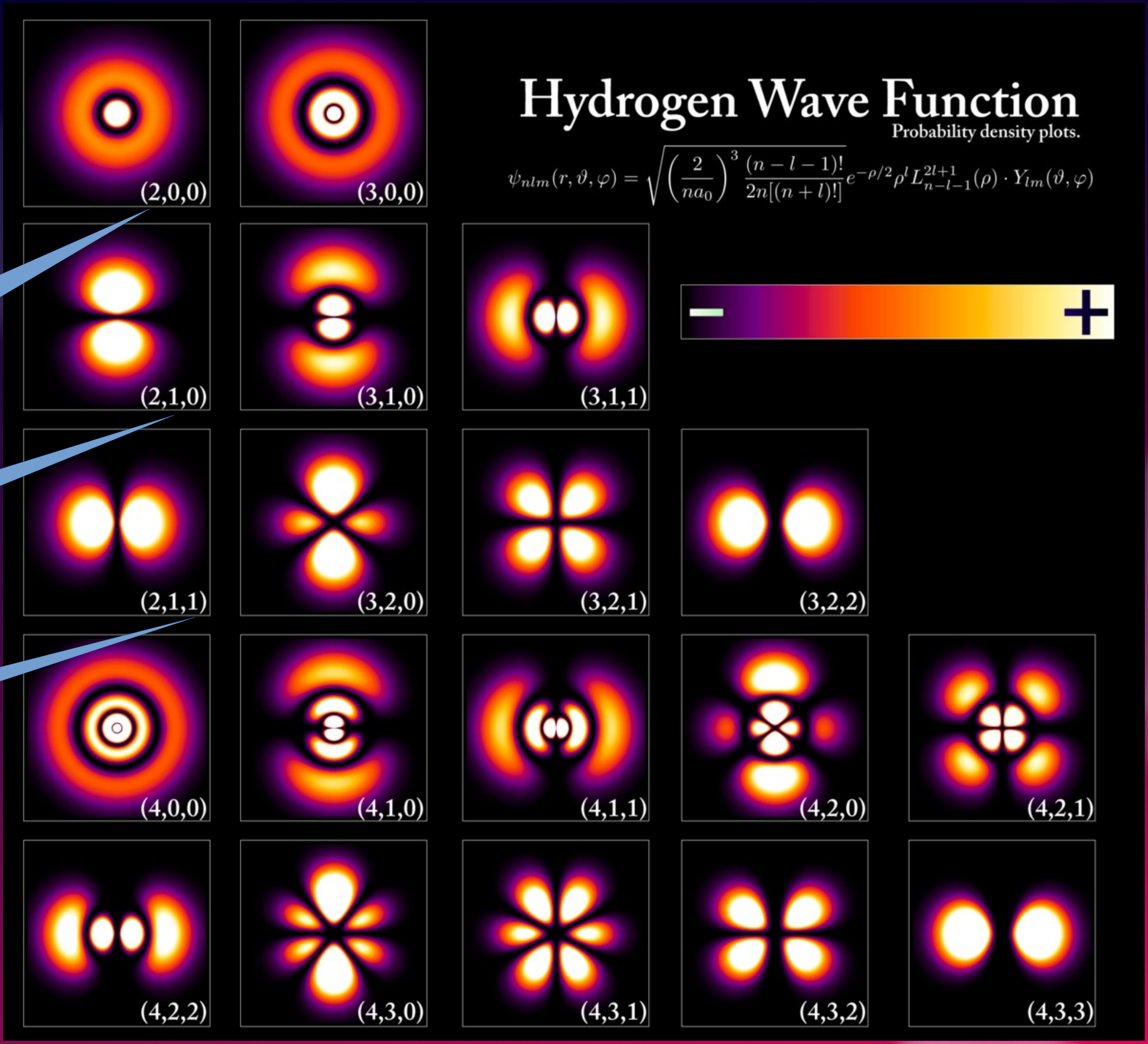
$$\mathbf{a}_{\text{rel}} := \mathbf{a}_1 - \mathbf{a}_2 = \left(1 + \frac{m_1}{m_2}\right)\mathbf{a}_1 = \frac{m_2 + m_1}{m_1 m_2} m_1 \mathbf{a}_1 = \frac{\mathbf{F}_{12}}{\mu}$$

Orbitals

"energy" *n*

"angular momentum" *l*

"z-ang. mom." *m*

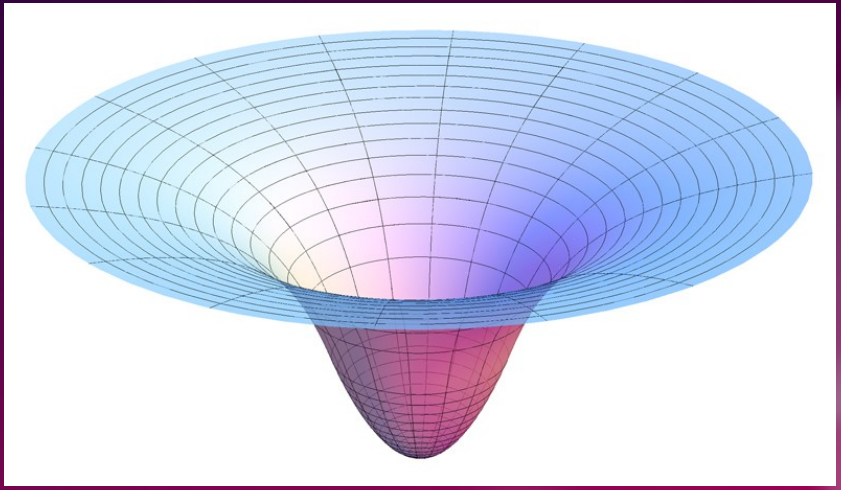


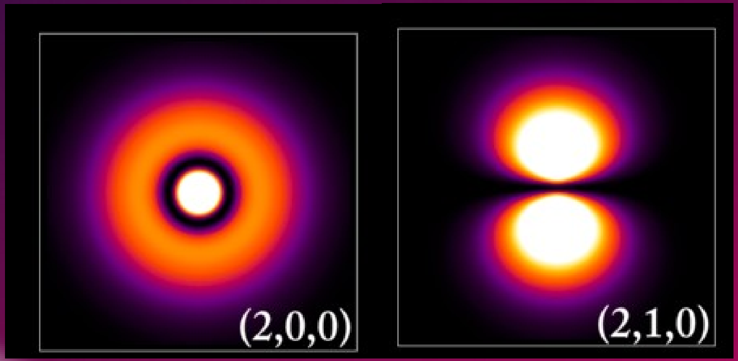
Fine structure

- Schrodinger gives same energy for different *l,m* values
- Relativistic correction $E_\mathrm{k} \approx \frac{p^2}{2m} - \frac{p^4}{8m^3c^2}$.
- Spin-orbit interaction $\mathcal{H}_\mathrm{SO} = \frac{1}{2}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)\left(\frac{g_s}{2m_e^2c^2}\right)\frac{\vec{L}\cdot\vec{S}}{r^3}$
  - Separates *m*=+/-1, but *m*=0 still degenerate between *l*s
- Lamb Shift (*please stand by*)
- Hyperfine (incorporates proton spin)

Lamb shift heuristically: proton radius

- V(r) ~ -1/r outside proton
- V(r) ~ constant inside
  - (depends on model)





Proton radius experiments

- Lamb shift (2003): 0.8768 ± 0.0069 fm
- Scattering (2000): 0.8775 ± 0.0005 fm
- Muonic hydrogen (2010): 0.842 ± 0.001 fm
  - ?????

Exotic atoms

- Hydrogen: proton + electron
- Muonic hydrogen: proton + negative
- Muonium: positive muon + electron
- Positronium: pos electron

Still pretty good
reduced mass!

Best
reduced mass!

But also
annihilation

Muonic hydrogen

- Reduced mass is ~200x higher
  - way smaller “atom”
- Proton is 1/500 rather than 1/100,000 of atom radius
- 100x more sensitive to proton radius
- Proton size 4% smaller than in hydrogen
- NEW PHYSICS????

Resolution: Nine years later

- Redoing the hydrogen experiments gives a value consistent with the muonic hydrogen experiment
- ¯_(ツ)_/¯ It's a hard experiment

Lamb shift, more precisely

- Correct treatment of the Lamb shift requires Quantum Electrodynamics (QED), a specific Quantum Field Theory (QFT)
- QFT originated by taking the number of particles to be an observable
- Uses the famous Feynman Diagrams

Feynman Diagrams

- “Feynman diagrams are purely symbolic; they do *not* represent particle trajectories.... The vertical dimension is *time* and horizontal spacings do *not* correspond to physical separations.... Each Feynman diagram actually stands for a particular *number*....” - Griffiths
- “In these diagrams, a line segment stands for a propagator $\frac{1}{i}\Delta(x-y)$, a filled circle at the end stands for a source $i\int d^4x J(x)$, and a vertex joining three line segments for $iZ_g g\int d^4x$.” - Srednicki

Feynman diagrams for Lamb shift

- from Griffiths, *Introduction to Elementary Particles*

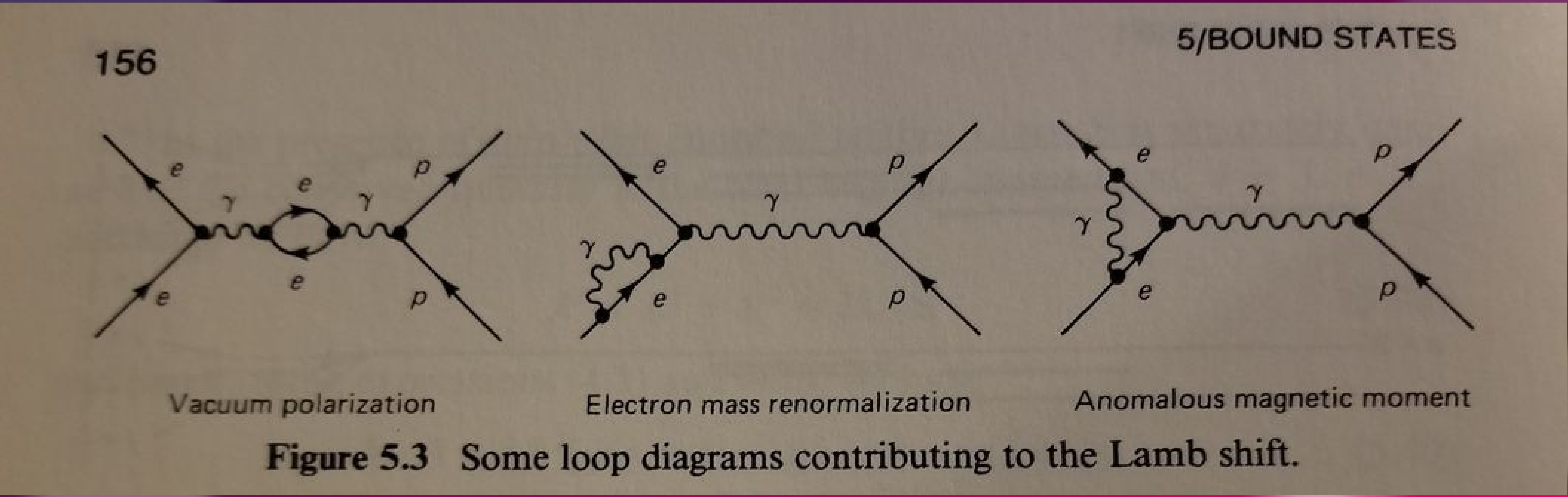
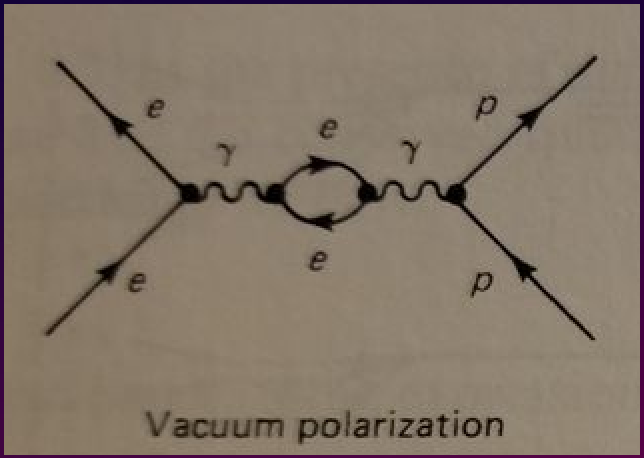


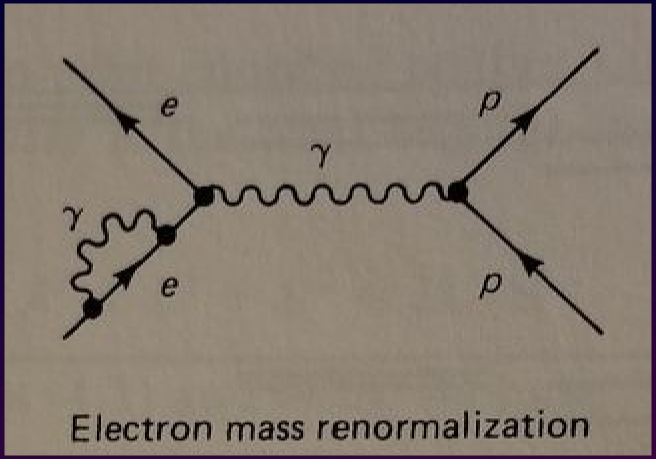
Figure 5.3 Some loop diagrams contributing to the Lamb shift.

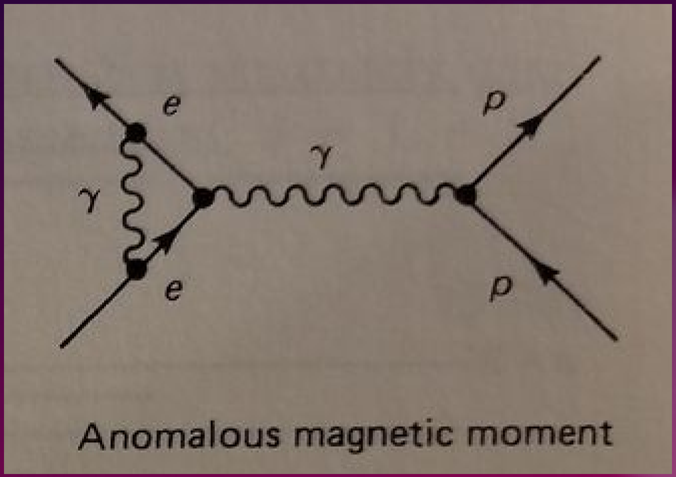
Vacuum polarization



- Hand-wavy explanation
  - vacuum is not really empty
  - temporary pair production
  - vacuum acts like a dielectric
  - super close to (inside) the proton, the attraction is weaker
  - (hint: proton radius)

Electron fuzzyness





- Apparent vs bare masses
- Electron self-interaction
- Point particles?
- Zitterbewegung
- A great quantitative success of QED

References

- Griffiths, *Introduction to Elementary Particles*
- Srednicki, *Quantum Field Theory*
- Gottfried & Yan, *Quantum Mechanics: Fundamentals*
- Pohl, *The size of the proton*, Nature, July 2010
- Wikipedia, copiously